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MODIFIED BETATRON STABILITY CALCULATIONS.(U)

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MODIFIED BETATRON STABILITY CALCULATIONS

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Cold beam linear growth rates of the negative mass and resistive wall instabilities in a high current modified betatron have been determined numerically. The resistive wall instability is much slower than the negative mass instability even for large wall resistivities. The two instabilities can be suppressed by a 10% electron energy spread provided $I(kA) < 2[B_\theta(kg)]^{1/3} \cdot (\gamma/10)^{5/2}$.

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MODIFIED BETATRON STABILITY CALCULATIONS

The Naval Research Laboratory presently is engaged in the point design of a high current modified betatron[1,2] in preparation for building such a device. This note summarizes linear growth rate computations of the negative mass and resistive wall instabilities performed in support of the design effort. Table 1 lists the base parameters assumed. In the following we first consider instability growth in a cold beam and then estimate reduction of the growth rates caused by a spread in beam particle energy.

Cold beam results were obtained by solving numerically the dispersion relation derived in Ref. 3. The dispersion relation predicts two distinct regimes of negative mass instability, one lying above and the other below a transition energy

$$\gamma_{tr} \approx (4\,\nu\, r_o^2/a^2)^{1/3} \quad . \tag{1}$$

For the parameters of Table 1 with I = 10 kA, $\gamma_{tr} \approx 6.2$. Figure 1 shows the growth rates versus beam energy above the transition point for the first three toroidal mode numbers. Wall conductivity is infinite, and I = 10 kA, B_θ = 5 kg. Note that the peak growth scales as $\ell^{1/2}$. We also ran infinite conductivity cases for I = 10 kA, B_θ = 1.46 kg and I = 2 kA, B_θ = 1.42 kg. Results for ℓ = 1, plotted in Figure 2, indicate a $(\gamma I/B^2)^{1/3}$ scaling of the growth rate for γ's below the peaks of the growth rate curves, in agreement with Ref. 3. The peak growth rate, on the other hand, scales roughly as $(I/B)^{1/2}$.

Behavior of the growth rate curves in the vicinity of the transition energy is illustrated in Figure 3 for ℓ = 1 and in Figure 4 for ℓ = 2; I = 10 kA, B_θ = 5 kg. The curves go to zero with infinite slope at γ_{tr}.

TABLE 1. NOMINAL MODIFIED BETATRON PARAMETERS

Major Radius	r_o = 100 cm	
Minor Radius	a = 10 cm	
Beam Radius	r_b = 1 cm	
Guide Field	$B_\theta \le 5$ kg	($\omega_\theta \le 2.933$ cm^{-1})
Vertical Field	B_z	($\omega_z = \beta\gamma/r_o$)
Field Index	n = 1/2	
Beam Current	I $\le$ 10 kA	($\nu < 0.588$)
Beam Energy	U = 2-50 MeV	(γ = 5-100)
Resistivity	250, 0 μΩ-cm	($\sigma = 1.5 \cdot 10^6$, ∞ cm^{-1})

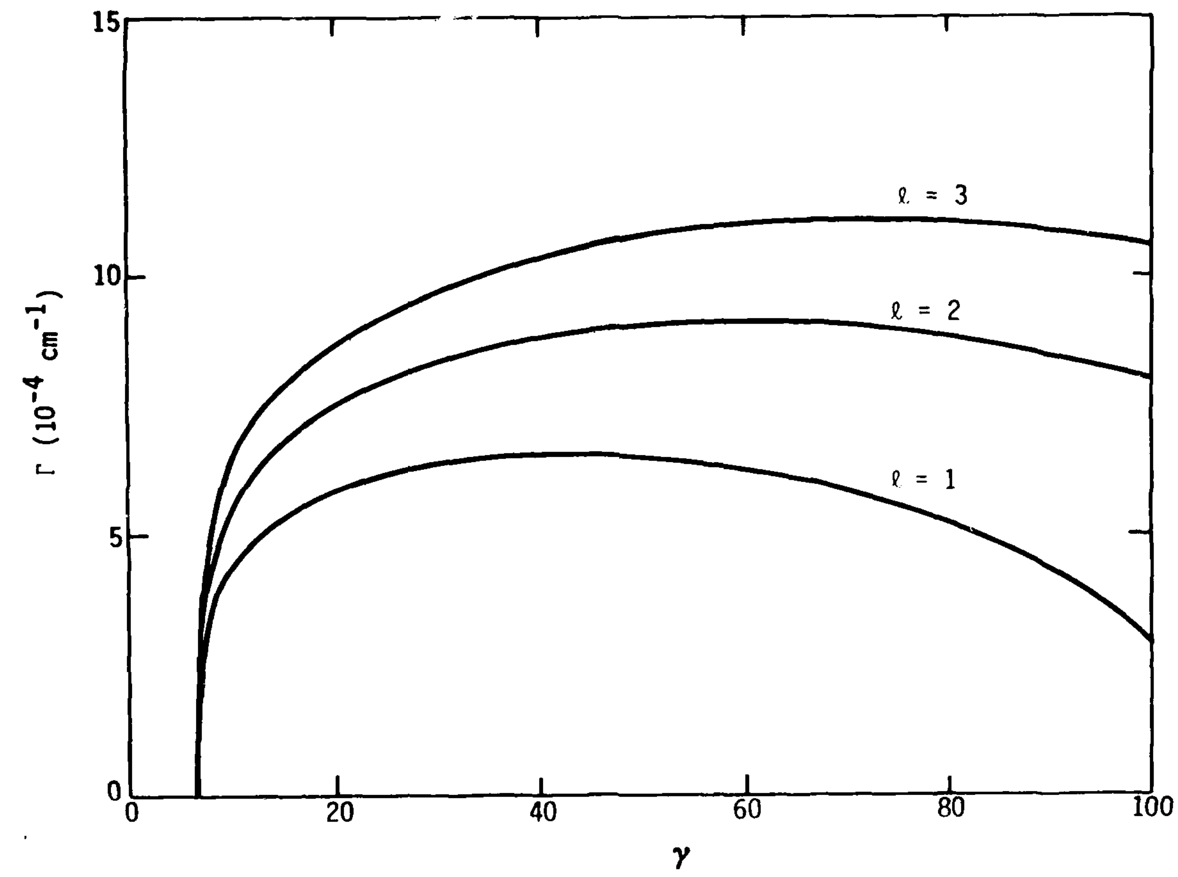

Figure 1. L = 1-3 negative mass instability growth rates versus beam energy above the transition point for I = 10 kA, B_θ = 5 kg.

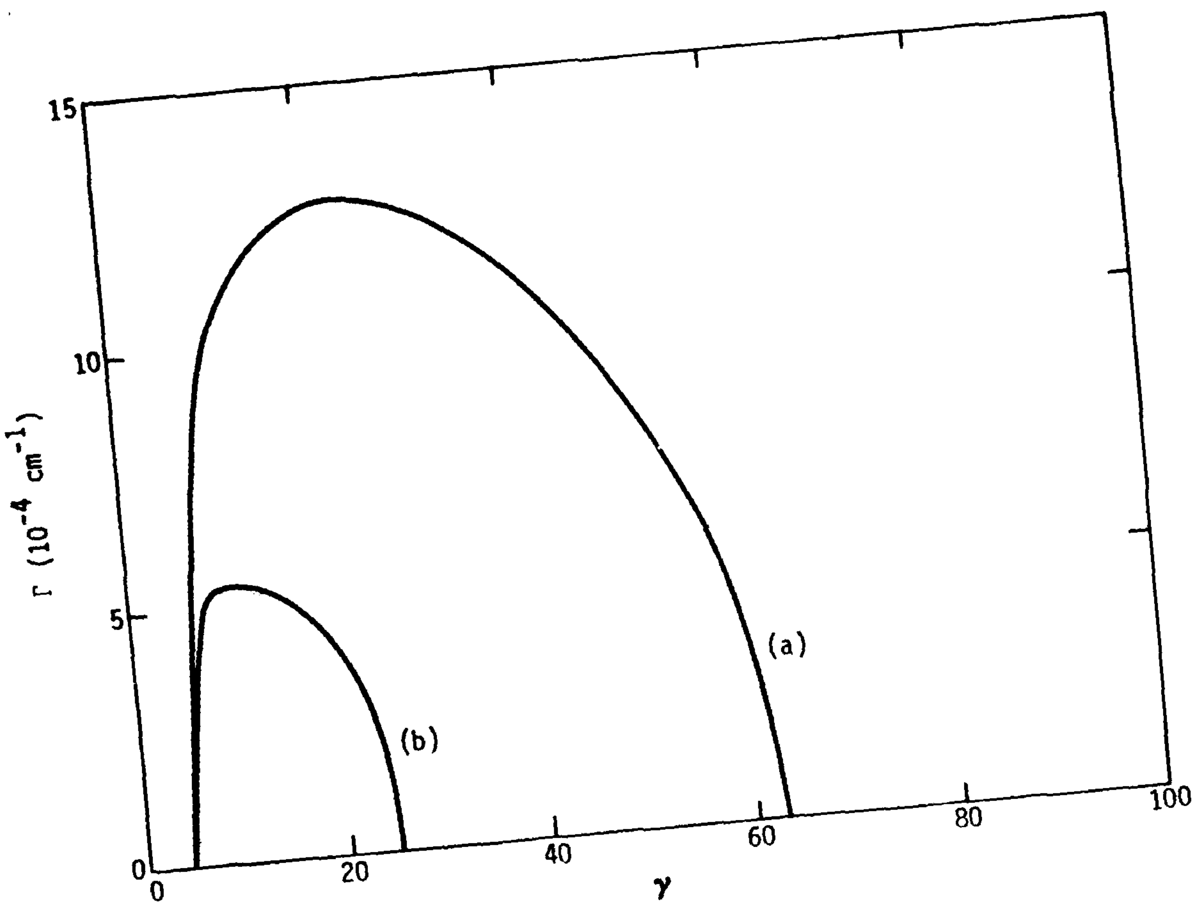

Figure 2. L = 1 negative mass instability growth rates versus beam energy above the transition point for (a) I = 10 kA, B_θ = 1.42 kg, and (b) I = 2 kA, B_θ = 1.42 kg.

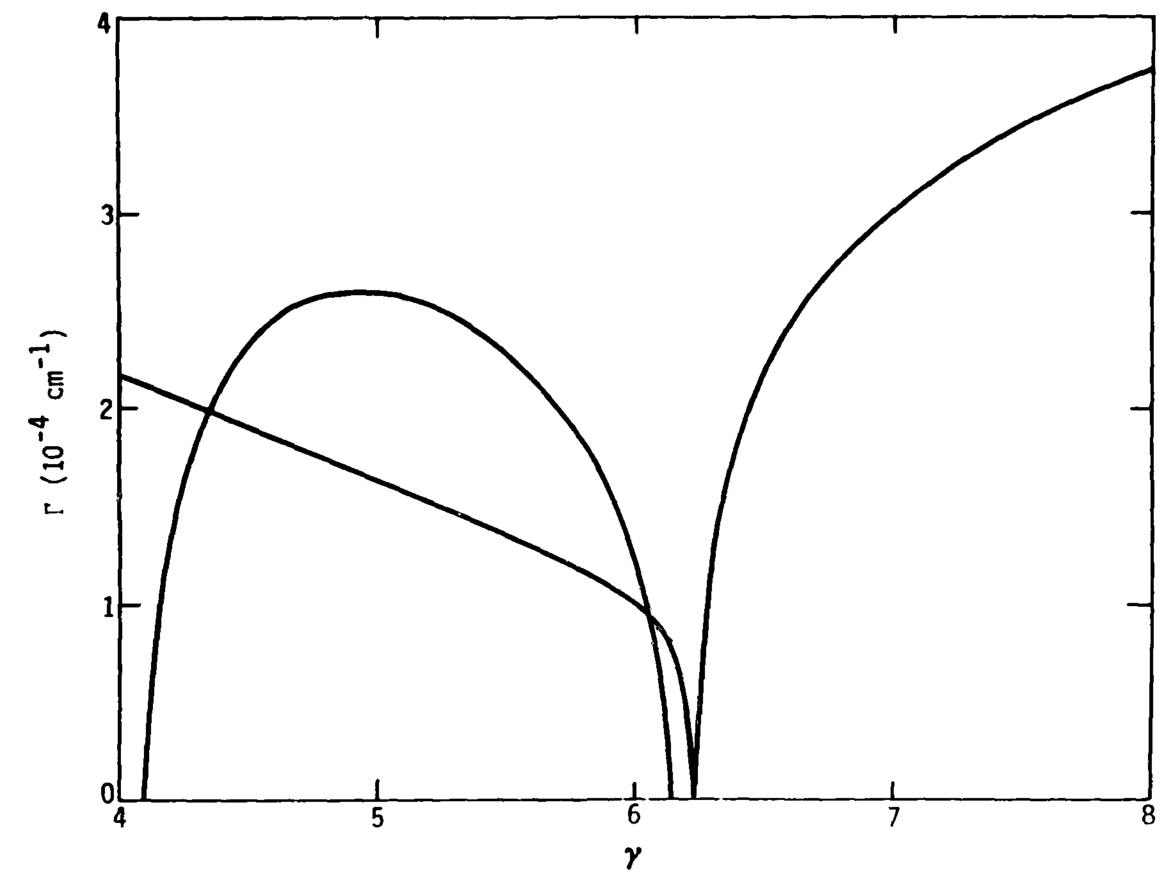

Figure 3. L = 1 negative mass instability growth rates versus beam energy in the vicinity of the transition point for I = 10 kA, B_θ = 5 kg.

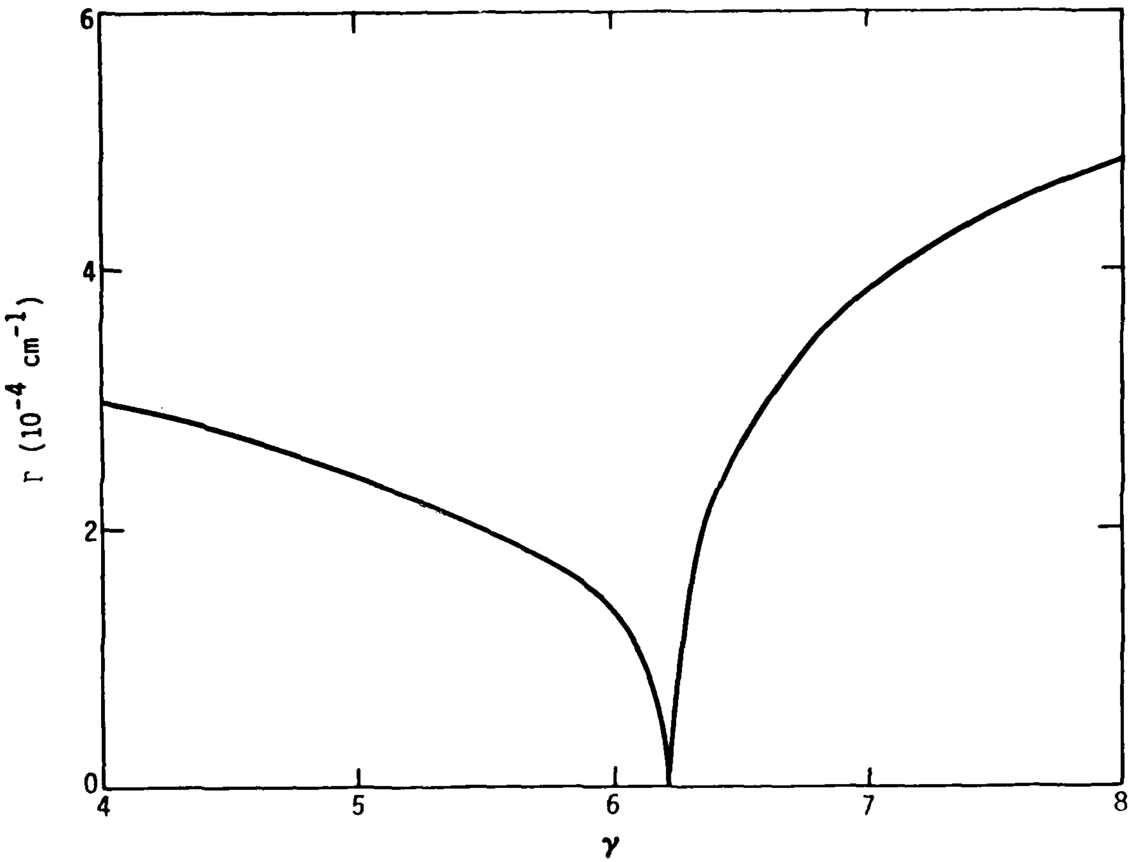

Figure 4. L = 2 negative mass instability growth rates versus beam energy in the vicinity of the transition point for I = 10 kA, B_θ = 5 kg.

Below γ_{tr} one or two unstable modes occur, depending on the parameters chosen. The growth rate dependence on ℓ in this region typically is weak, while the scaling with current and toroidal magnetic field crudely follows the expressions valid above γ_{tr}.

Introducing a 250 $\mu\Omega$-cm wall resistivity changes the negative mass growth curves of Figures 1 through 4 by only a few percent except in the immediate vicinity of γ_{tr}, where the otherwise occurring sharp dip in the growth rate curve is partially filled in. Additionally, resistivity drives unstable a previously stable negative mass mode but with a growth rate down from those of the original modes by an order of magnitude. Compare Figure 5 to Figure 3.

A transverse cyclotron wave instability also is triggered by wall resistivity, when $\omega \approx \ell/R - \omega_\theta/\gamma > 0$. Figure 6 depicts growth rates for ℓ = 4, 8 at I = 10 kA, B_z = 5 kg. As noted in Ref. 4, this instability is well described by models developed for beams in straight drift tubes. In particular, the peak growth rate formula

$$\Gamma = \frac{1}{2} \sigma^{-1/3} \left(\frac{\omega_p^2 r_b^2}{\omega_\theta a^3}\right)^{2/3} \qquad (2)$$

reproduces data in Figure 6 to within 5%. (The conductivity σ is measured in cm^{-1} and contains a factor 4π.) The transverse resistive wall instability is seen to be somewhat slower than the negative mass instability even for the rather high resistivity assumed. Moreover, the growth curves are much narrower in γ. We conclude that wall resistivity is not a significant factor in modified betatron stability.

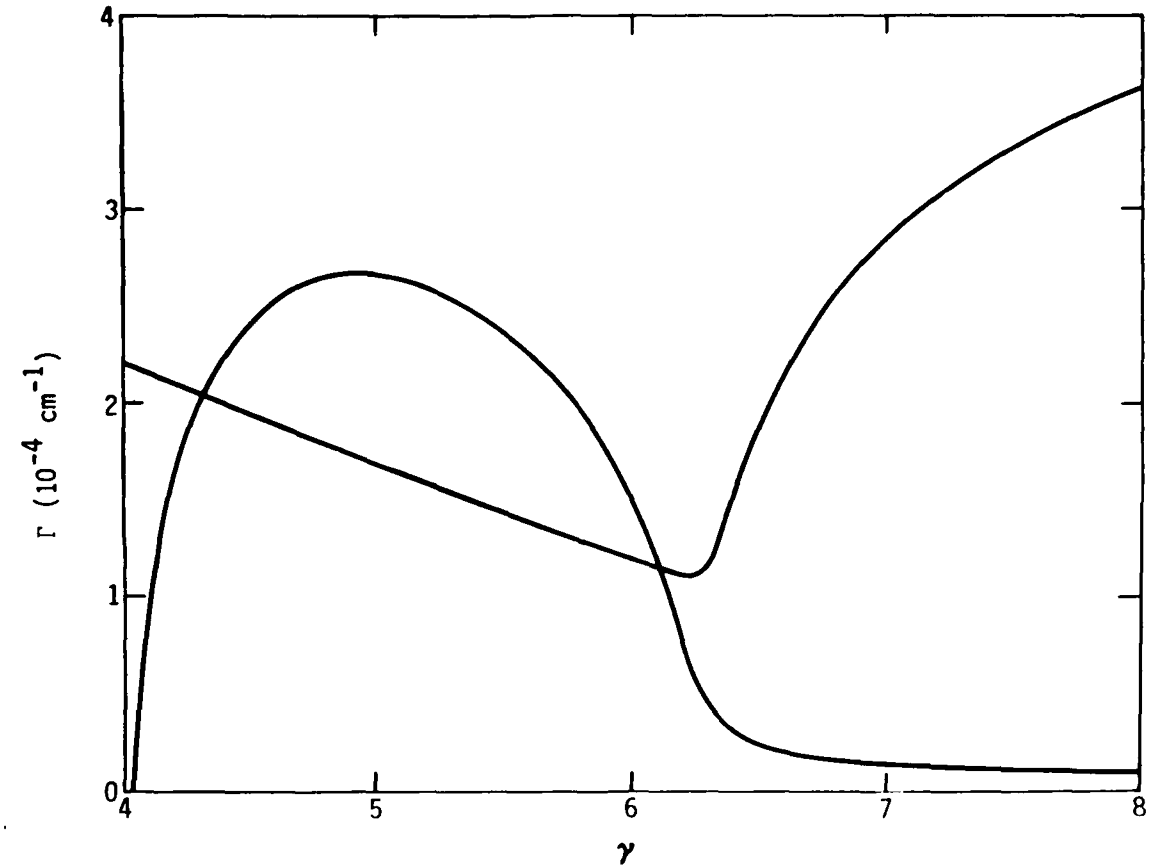

Figure 5. L = 1 negative mass instability growth rates, as modified by 250 μΩ-cm wall resistivity, versus beam energy in the vicinity of the transition point for I = 10 kA, B_θ = 5 kg.

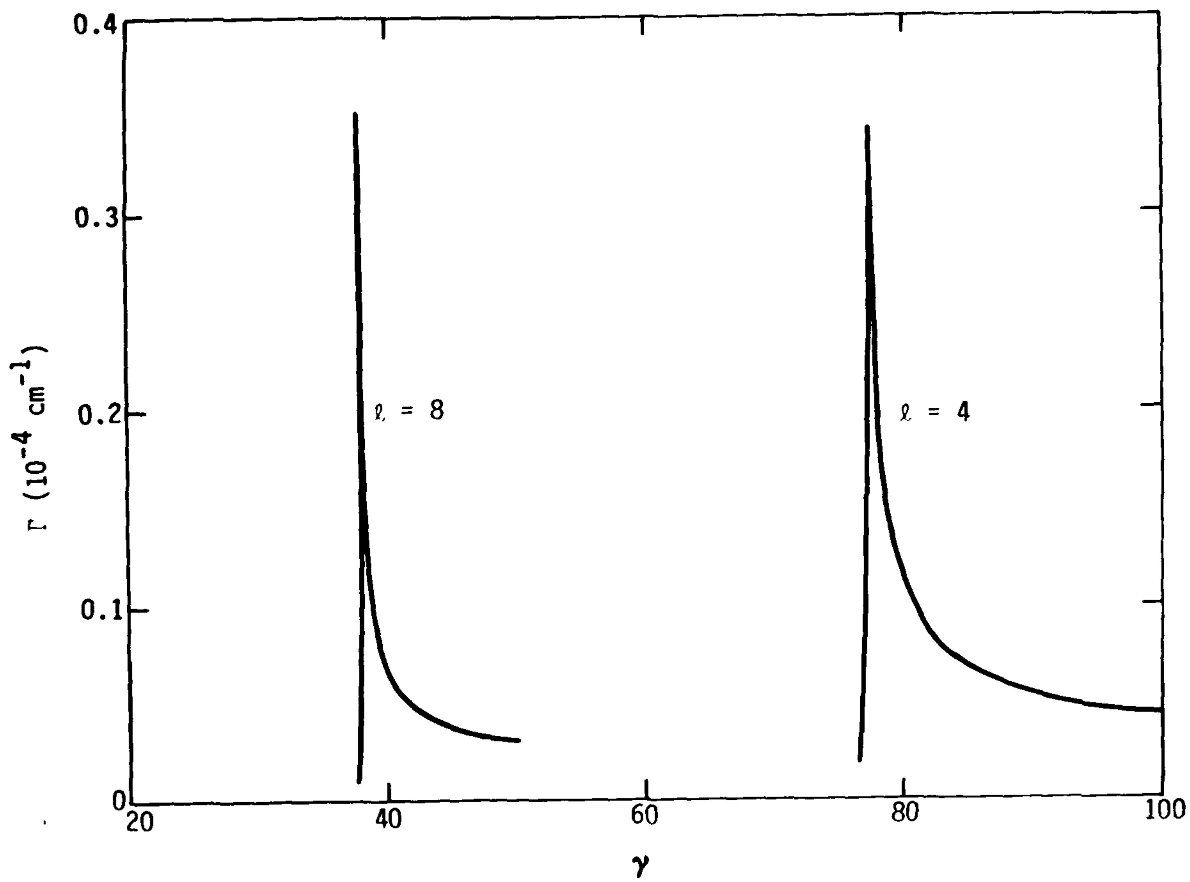

Figure 6. L = 4, 8 transverse resistive wal' growth rates, due to 250 μΩ-cm wall resistivity, ve ...gy for I = 10 kA, B_θ = 5 kg.

Table 2 summarizes peak growth rates for the various cases discussed above. In view of the long acceleration times required (~ 10^7 cm), thermal effects must be involved to ameliorate the instabilites.

Reference 5 estimates the (Lorentzian) electron energy spread necessary to suppress an instability of growth rate Γ as

$$\Delta\gamma \geq \Gamma/\ell r_o |k| \tag{3}$$

with

$$k \equiv [\gamma^{-2} + (n_s + n - 1)^{-1}]/\gamma r_o^2 \tag{4}$$

and

$$n_s = n_o/2\gamma\omega_z^2 \approx 2\nu r_o^2/r_b^2\gamma^3 \ . \tag{5}$$

For parameters of interest here, the second term in Eq. (4) dominates, giving

$$\Delta\gamma/\gamma \geq (\Gamma r_o/\ell)|n_s - 1/2| \ . \tag{6}$$

Since Γ increases with ℓ as $\ell^{1/2}$ or slower, the greatest energy spread is required for $\ell = 1$. Furthermore, the γ^{-3} dependence of n_s and the weak γ dependence of Γ dictate evaluating $\Delta\gamma$ for $\gamma \sim 5$-10, the worst case.

From Figure 1 and Table 1, evaluating $\Delta\gamma$ at $\gamma = 10$ yields $\Delta\gamma/\gamma \approx 0.5$, which is unacceptably large. The corresponding spread for $\gamma = 5$ is, of course, several times worse. To obtain a more realistic energy spread, say

TABLE 2. INSTABILITY PEAK GROWTH RATES

CASE	Γ (10^{-4} cm^{-1})
10 kA, 5 kg, $\sigma = \infty$	
ℓ = 1 negative mass	6.57
ℓ = 2 negative mass	9.13
ℓ = 3 negative mass	11.1
10 kA, 1.42 kg, $\sigma = \infty$	
ℓ = 1 negative mass	12.7
2 kA, 1.42 kg, $\sigma = \infty$	
ℓ = 1 negative mass	5.41
10 kA, 5 kg, $\sigma = 1.5 \cdot 10^6$ cm^{-1}	
ℓ = 4 resistive wall	0.35
ℓ = 8 resistive wall	0.36

TABLE 3. COMPUTED MAXIMUM CURRENT FOR WHICH 10% ENERGY SPREAD STABILIZES INSTABILITIES

B_z (kg)	γ	I (kA)
5.0	10	3.2
1.42	10	2.0
5.0	5	0.6
1.42	5	0.4

0.1, we recall that n_s scales as I/γ^3 and Γ as $(\gamma I/B_\theta^2)^{1/3}$. Combining these dependencies yields a semi-empirical upper bound on the beam current for which a 10% energy spread damps out the negative mass and resistive wall instabilities,

$$I(kA) < 2\ [B_\theta(kg)]^{1/3}\ (\gamma/10)^{5/2}\ . \qquad (7)$$

The exponent of B_θ in (7) was adjusted from 1/2 to 1/3 to agree better with the numerical data of Table 3.

In summary, we have determined the cold beam linear growth rates of the negative mass and resistive wall instabilities for a modified betatron with dimensions and guide fields listed in Table 1. The resistive wall instability was found to be relatively insignificant. The negative mass instability, on the other hand, probably would disrupt the beam during its long acceleration time unless suppressed by electron thermal spread. A model based on a Lorentzian distribution of particle energies predicts that a 10% spread in energy is sufficient to eliminate the negative mass instability provided the beam current satisfies Eq. (7). Since this inequality is quite restrictive for small γ, every effort should be made to have the beam injection energy as high as possible. Increasing the toroidal guide field also helps. Not considered here are transverse betatron oscillations, which also should have a stabilizing effect.

These calculations were suggested by Phillip Sprangle of the Naval Research Laboratory and were supported by the Office of Naval Research.

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